



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

BOOK REVIEWS

Monographs on Topics of Modern Mathematics Relevant to the Elementary Field.

Edited by J. W. A. YOUNG. New York: Longmans, Green & Co.,
1911. Pp. vii+416.

An extract from the editor's preface will give a good idea of the scope and purpose of this work:

"Among the various publications on mathematics that are being made, it would seem that there is room for a serious effort to bring within reach of secondary teachers (in service or in training), college students, and others at a like stage of mathematical advancement, a scientific treatment of some of the regions of advanced mathematics that have points of contact with the elementary field. Undoubtedly one of the most crying needs of our secondary instruction in mathematics today is that the scientific attainments of the teachers be enlarged and their mathematical horizon widened; and I believe that there is a large body of earnest teachers and students that are eager to extend their mathematical knowledge if the path can be made plain and feasible for them.

"A volume of monographs dealing with selected topics of higher mathematics might well be a useful contribution to the meeting of this need. Such monographs would aim to bring the reader into touch with some characteristic results and viewpoints of the topics considered, and to point out their bearing on elementary mathematics. They would therefore contain:

"(1) A considerable body of results proved in full, so that the reader can materially extend his mathematical acquisitions by the reading of the monograph alone.

"(2) Statement without proof of some leading methods and results, so as to give a bird's-eye view of the subject.

"(3) A small number of references indicating what the reader may profitably take up after he has mastered the contents of the monograph.

"Each author retains sole responsibility for his monograph as it now appears. No attempt has been made to secure uniformity in style of treatment; each monograph is an independent unit, that can be read without reference to the others.

"The amount of technical mathematical knowledge that is presupposed on the part of the reader varies with the different subjects. A large part of the book presupposes only knowledge of elementary geometry and algebra, together with a certain measure of mathematical maturity. On the other hand, there is much that will repay careful and detailed study by advanced students. So far as the subject-matter permits, the less difficult topics are taken up first in each monograph."

The nine monographs in this collection are:

- I. "The Foundations of Geometry," by Oswald Veblen.
- II. "Modern Pure Geometry," by Thomas F. Holgate.
- III. "Non-Euclidean Geometry," by Frederick S. Woods.
- IV. "The Fundamental Propositions of Algebra," by Edward V. Huntington.
- V. "The Algebraic Equation," by G. A. Miller.

VI. "The Function Concept and the Fundamental Notions of the Calculus," by Gilbert Ames Bliss.

VII. "The Theory of Numbers," by J. W. A. Young.

VIII. "Constructions with Ruler and Compasses; Regular Polygons," by L. E. Dickson.

IX. "The History and Transcendence of π ," by David Eugene Smith.

The authors are all professors of mathematics in American universities, and well known as writers in their special fields. Their names alone are a sufficient guaranty of the quality of this work. Not only has their judgment in choice of material been excellent and their work clear-cut and accurate, but they have in the main succeeded in making their presentation really readable to those for whom it was intended (a by-no-means easy accomplishment for the specialist). To select but a single instance, few will commence Professor Bliss's paper without finishing it, if possible, at one sitting; to the reviewer, at least, it was as fascinating as a novel; and yet this is in the supposedly arid region of function-theory and the calculus.

A glance at the list of titles shows that there is no attempt here at a general introduction to advanced mathematics. All the monographs are in the field of geometry and algebra alone, except VI, VII, and IX. Applied mathematics is scarcely referred to, and only the barest elements of the calculus are presented. The choice of material has evidently been determined largely by the two considerations of interest and accessibility to the audience addressed. None of the papers is, however, a "mathematics made easy." The spirit of criticism that characterizes the present period is dominant; theorems are stated accurately, and not "in general"; what is proved and what is assumed are clearly distinguished. And, almost uniformly, each article leaves one impressed with the magnitude of the field to which it leads, and curious to know more of some of the details merely mentioned or hinted at.

As the editor has stated, this book is intended both for secondary teachers and for college students. That the list of those who may profit by its reading is by no means so circumscribed is evident to the reviewer from his own experience. However, if we turn to the audience more especially addressed, it may be asked whether, after all, the material is really within reach of secondary teachers, especially of those whose daily work has left little opportunity for keeping fresh their college mathematics. Certainly previous attempts, such, for instance, as have been made to popularize the calculus, have not been unqualified successes. It is only a partial answer to say that none of the articles presupposes much more than the ordinary Freshman course in mathematics, and some not even as much as this. Monograph I presupposes only the most general principles of logic, yet many will find it the most difficult of the collection to follow in detail. It may be safely asserted that "skimming" most of these papers will yield little except to one thoroughly familiar with the material, but that very few will fail to profit by a careful reading of any one article. And there should prove to be no real obstacle to the mastering of any proof actually carried through here, *provided* the reader is willing to put in the necessary thought and care.

As to the other division of the audience addressed, advanced college students, there can be no difference of opinion regarding the service rendered. This book should be placed in every seminar and mathematical reading-room. It will be a potent factor in many a mathematical awakening.

Any extended consideration of the scope or method of each paper would, perhaps, be out of place here. Brief summaries will suffice.

The first three monographs may be grouped together, as they treat of the same field, geometry, though by no means with the same method. Professor Veblen's monograph develops the foundations of geometry, starting with but one undefined element, the point, and two undefined relations, order and congruence. This is carried out with full detail far enough to cover the ground of Euclid's first twenty-eight propositions. The remainder of the paper is merely an outline. Considerations of independence, consistency, and sufficiency are not entered upon. The next paper is in strong contrast. It makes little attempt at exactness of definition, but lays its main stress on the obtaining of results. Euclid is assumed. The author is thus enabled in thirty pages to go as far as Pascal's and Brianchon's theorems, a discussion of conic sections, and an introduction to poles and polars, mainly but not entirely by synthetic methods. In Professor Woods's monograph the style and treatment lie between those of the two preceding. For its foundation one may use either Professor Veblen's exact system, or Euclid's less complete one, up to the parallel postulate. No little skill is shown in bringing together the points of view of Lobachevsky and Bolyai, of Riemann, and of Cayley and Klein. There is more detail here than in most of the other papers, and it will not be found the easiest to read, but on the other hand it gives a more complete introduction to its field than do most of the others.

The building-up of a set of postulates for any branch of mathematics, perhaps we should say more exactly for any mathematical system, is often considered one of the driest of pursuits. Professor Huntington's paper on the "Fundamental Propositions of Algebra" is, however, one of the most interesting of this collection. How he makes it so may be left to the reader. It is much simplified by first considering concrete examples of the abstract theory. Questions of consistency, sufficiency, and independence are considered in detail. In one of the appendices is given a proof that every algebraic equation has a root in which the function-theoretic details seem to be simplified to the minimum.

Monographs V and VI attempt, for obvious reasons, only a most general survey of their field. They are interesting and suggestive; their main function is not to convey detailed information, but to point the way. In VI there is a rather annoying exception to the usually careful proofreading. But here, and in one or two places elsewhere, the twisted words and incorrect formulas can be set right without great trouble.

In VII a more detailed treatment is to be found. The classical theory of numbers is developed up to and including a brief treatment of binomial and quadratic congruences. The perennial fascination of the subject can hardly fail to be aroused in the reader. In VIII is developed the most interesting application of number theory and algebra. The author has succeeded in not spoiling its interest by a too-formidable massing of theorems and proofs; this is in part accomplished by deferring some of the proofs. This is probably the most readable presentation of the subject to be found anywhere, and is given with fairly full detail.

Monograph IX is another special application of subjects previously considered. Its first half is descriptive and historical, in Professor Smith's well-known style. The last part is devoted to Gordan's proof of the transcendence of e and π . This will be found by most the hardest nut to crack of the whole book, in spite of the limitation in stating proofs to equations of the third degree. The mixture of formulas at the end is unfortunate.

This book has no competitor in its field. Nothing of just this kind has hitherto appeared in English, or in any other language so far as the reviewer is aware. A need for it has existed, and much has here been done toward filling that need.

D. R. CURTISS

NORTHWESTERN UNIVERSITY

Internationales auf dem Gebiet der Erziehung. VON WILHELM MÜNCH.
(Internationale Wochenschrift für Wissenschaft, Kunst, und Technik,
March 4, 1911.)

This article reviews the present educational situation in France, England, the United States, and Germany. The review is "international" not only in that it includes a consideration of several countries but also in the fact that the different nations are compared with one another and that the criticisms, favorable and unfavorable, which are cited have been made in each case by foreign critics as well as by those at home. The outstanding criticisms which are brought to bear upon the French system are directed toward its scholasticism, emphasis upon rhetorical training, and intellectualism as contrasted with training of the will. Movements for reform are noticed, however, and it is recognized that a type of training which would not suit another people might be the best for the French. English education is criticized for its neglect of adequate scientific preparation and its accompanying overemphasis upon sport as a means of discipline of the will. At the same time the advantage in sympathetic relations between master and pupil and in freshness and vigor of judgment and action is recognized. A further advantage of the English system is its decentralization, which allows greater flexibility and presents less resistance to reform than a more closely organized system. The chief criticism of American education in foreign quarters seems to concern itself with the freedom which is allowed the pupils, particularly in choice of studies. The author finds that Germany does not hold the position of recognized leader in education which it once held. Foreigners are finding much to criticize in the hard-and-fast and centralized organization of the German system, the spirit of subjection to authority which is engendered, and the failure to encourage the development of individuality. The author finds an interesting historical reason for this situation in the fact that "we Germans are by nature more individualistic and therefore need for our common life a hand to control and bind us together more than many other nations." What is good for one nation may not be good for another. Nevertheless, German education, as well as that of other nations, is moving in the direction of reform. The value of international acquaintanceship and criticism appears in the tendency of each nation to adopt the good features of the systems of its neighbors.

FRANK N. FREEMAN

THE SCHOOL OF EDUCATION
THE UNIVERSITY OF CHICAGO